A Grammar Model of Routing in P2P Systems

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Outline

0. **The problem**
   – Infrastructure for Dynamic Binding of Host Identities
   – Properties to achieve

1. **Routing grammar**
   – Formal representation of node's routing rules
   – Scenarios: flows of messages
   – Routes = Derivations = Set of paths in the network

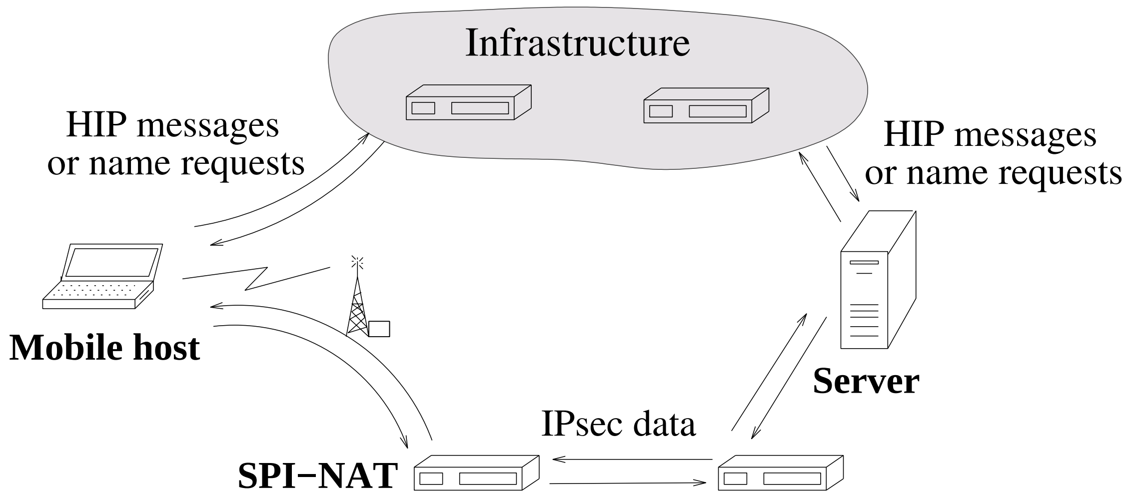
2. **Linear Diophantine model of routing**
   – Equations to describe routes
   – Basic routes and Hilbert basis

3. **Application and deployment**
   – connectivity&topology, efficiency, reliability, maintenance

The problem: scope

Dynamic binding of host identities

- requesting recipient's IP (HIT↔IPs mapping)
- sending HIP control messages to the recipient (forwarding)



Environment is dynamic, chaotic, and heterogeneous

- Node joins and leaving
- Node failures as well as links in underlying network
- untrusted parties
- ...

The problem: properties

Connectivity: any two peers are connected by a path
(existing DHTs work well but non-transitivity can happen)

Scalability: many end-hosts, many nodes
(at least Chord works well for i3 and Hi3)

Efficiency: small latencies of paths
(open problem)

Reliability: detecting failures and recovering, extra guarantee for delivering, resistance to attacks
(open problem)

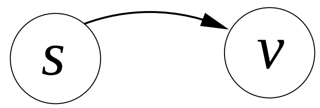
Low maintenance cost: control traffic and nodes CPU time
(open problem)

Routing grammar: rules

Infrastructure $S = \{s_1, s_2, \dots, s_N\}$.

T_s is a local routing table of node $s \in S$.

Base routing with retransmissions:

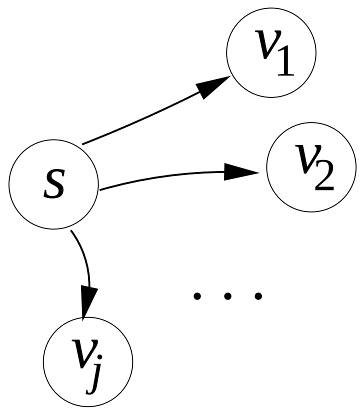


$\boxed{s \longrightarrow v^{k_v}}$, $v \in T_s$

node s forwards a message to v,

k_v is #attempts

Sequential and parallel forwarding:

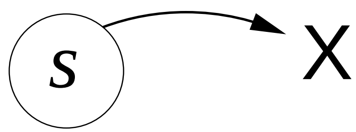


$\boxed{s \longrightarrow v_1^{k_1} v_2^{k_2} \cdots v_j^{k_j}}$, $\{v_1, v_2, \dots, v_j\} \subset T_s$

node s use alternate directions,

k_i is #attempts for each direction

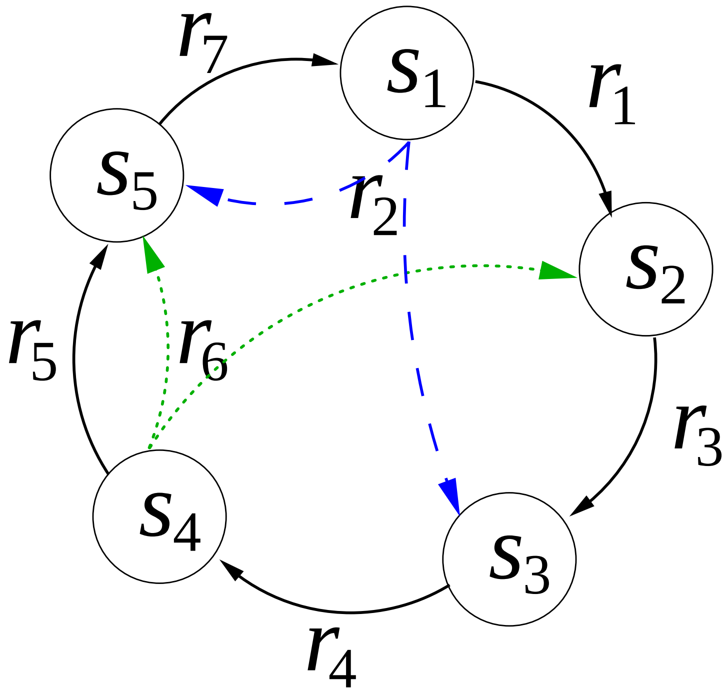
Completion of path: $\boxed{s \longrightarrow \varepsilon}$



node s does not forward a message further

Routing grammar: example 1

- Nodes $S = \{s_1, s_2, \ldots, s_5\}$
- Clockwise links (ring): r_1, r_3, r_4, r_5, r_7
- Parallel: r_2;   Sequential: r_6



$$
\begin{aligned}
r_1, r_2 :&\quad s_1 \longrightarrow s_2 \mid \mathrm{c} s_3 s_5 \\
r_3 :&\quad s_2 \longrightarrow s_3 \\
r_4 :&\quad s_3 \longrightarrow s_4 \\
r_5, r_6 :&\quad s_4 \longrightarrow s_5 \mid \mathrm{b} s_2 s_5 \\
r_7 :&\quad s_5 \longrightarrow s_1
\end{aligned}
$$

Terminals b and c characterize sequential and parallel forwarding, respectively

Routing grammar: paths and routes

Source nodes: each node $s \in S$ initiates b_s^- messages

Routing: messages flow by various paths,
some are duplicated, some are lost, …

Destination nodes: each node $v \in S$ receives b_v^- messages

Extra routing attributes:
b_σ is the total number of routing rules applied with attribute $\sigma \in \Sigma$

Derivation in the routing grammar:

$$\big(s^{b_s^-}\big)_{s\in S} \quad \Rightarrow^* \quad \big(\sigma^{b_\sigma}\big)_{\sigma\in\Sigma}\big(s^{b_s^+}\big)_{s\in S}$$

Route: all paths that messages used, $\quad (b^-) \xrightarrow{b} (b^+)$

set of paths = route = derivation

Several paths are considered simultaneously (aggregation)!

Routing grammar: example 2

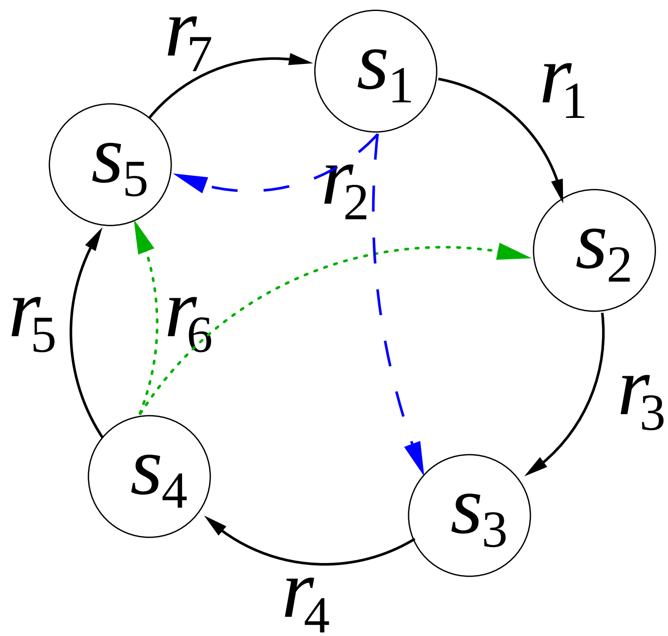
$b_1^- = b_1^+ = 1$: Circuit

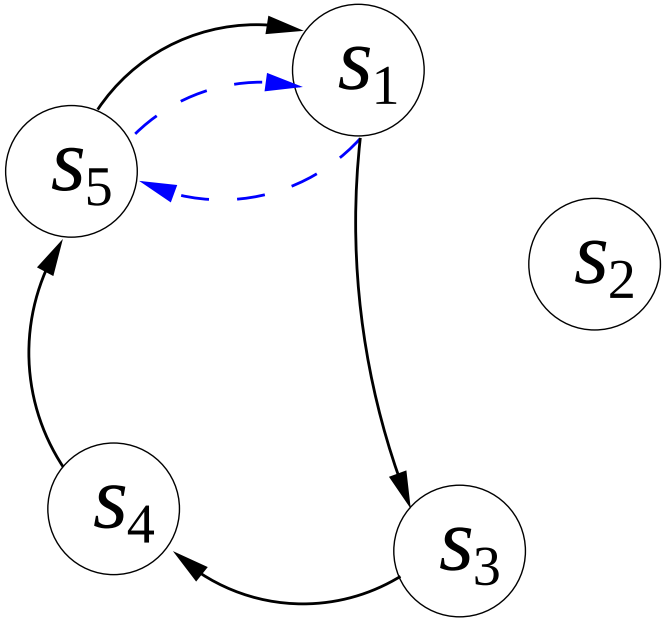
$$s_1 \stackrel{r_1}{\Rightarrow} s_2 \stackrel{r_3}{\Rightarrow} s_3 \stackrel{r_4}{\Rightarrow} s_4 \stackrel{r_5}{\Rightarrow} s_5 \stackrel{r_7}{\Rightarrow} s_1$$

$b_1^- = 1,\ b_1^+ = 2$: Duplication

$$s_1 \stackrel{r_2}{\Rightarrow} \mathrm{c} s_3 s_5 \stackrel{r_4}{\Rightarrow} \mathrm{c} s_4 s_5 \stackrel{r_5}{\Rightarrow}$$

$$\Rightarrow \mathrm{c} s_5^2 \stackrel{r_7}{\Rightarrow} \mathrm{c} s_1 s_5 \stackrel{r_7}{\Rightarrow} \mathrm{c} s_1^2$$





Two paths in the cyclic route

Linear Diophantine model: construction

- $|S| = N$, $|R| = m$, $|\Sigma| = t$, $n = N + t$

- Non-negative linear Diophantine equations associated with a routing grammar (ANLDE system):

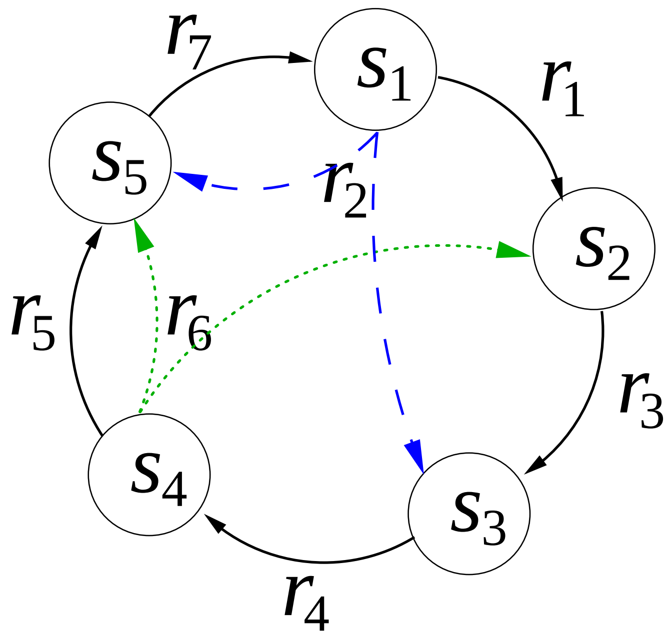
$$\begin{cases} \displaystyle\sum_{r\in R_s} x_r + b_s^+ = \sum_{r\in R} a_{sr} x_r + b_s^- \ , & s \in S \\ \displaystyle\sum_{r\in R} a_{\sigma r} x_r = b_\sigma \ , & \sigma \in \Sigma \end{cases}$$

- Model of routes $(b^-) \xrightarrow{b} (b^+)$ (P2P flows)

Linear Diophantine model: example 1

Circuits: $(\mathbf{0}) \longrightarrow (\mathbf{0})$ or $s \Rightarrow^* s$

$$\begin{cases} s_1 : & x_1 + x_2 = x_7 \\ s_2 : & x_3 = x_1 + x_6 \\ s_3 : & x_4 = x_2 + x_3 \\ s_4 : & x_5 + x_6 = x_4 \\ s_5 : & x_7 = x_2 + x_5 + x_6 \end{cases}$$

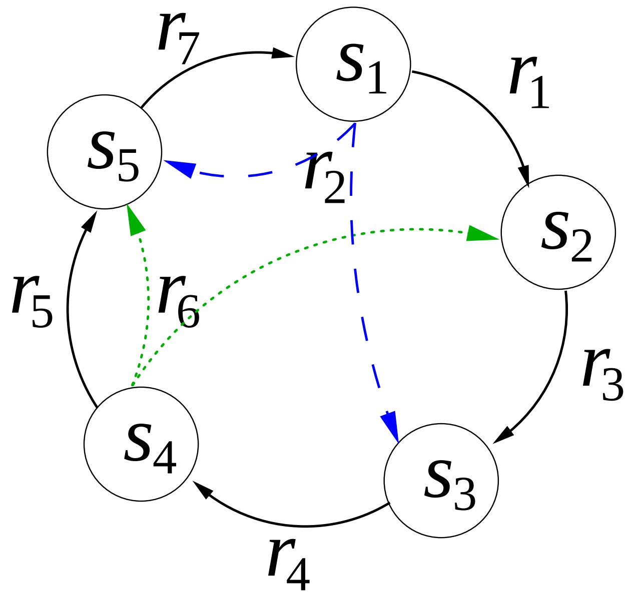


Basic solutions:

$$x = (1, 0, 1, 1, 1, 0, 1)^{\mathsf{T}}$$

$$s_1 \overset{r_1}{\Rightarrow} s_2 \overset{r_3}{\Rightarrow} s_3 \overset{r_4}{\Rightarrow} s_4 \overset{r_5}{\Rightarrow} s_5 \overset{r_7}{\Rightarrow} s_1$$

Linear Diophantine model: example 2



Doubling routes from s_1: $s_1 \Rightarrow^* s_1^2$

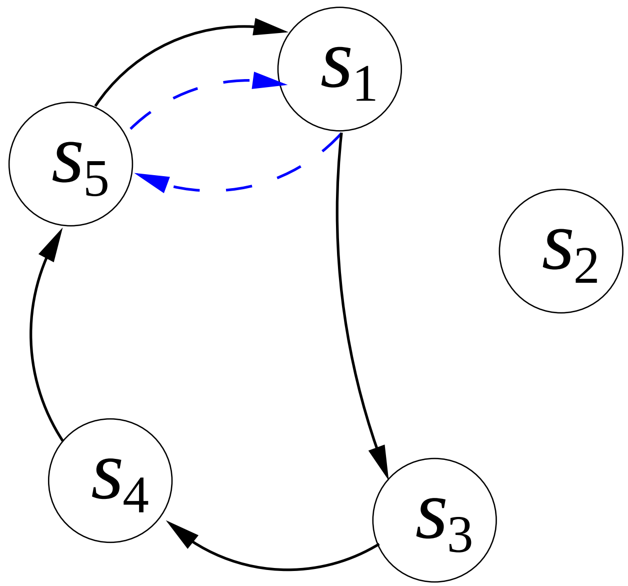
$$b^- = (1,0,0,0,0),\ b^+ = (2,0,0,0,0)$$

$$\begin{cases} s_1 : & x_1 + x_2 + 2 = x_7 + 1 \\ s_2 : & x_3 = x_1 + x_6 \\ s_3 : & x_4 = x_2 + x_3 \\ s_4 : & x_5 + x_6 = x_4 \\ s_5 : & x_7 = x_2 + x_5 + x_6 \end{cases}$$

Basic solutions:

$$x = (0,1,0,1,1,0,2)^{\mathsf{T}}$$

$$s_1 \xRightarrow{r_2} s_3 s_5 \xRightarrow{r_4} s_4 s_5 \xRightarrow{r_5} s_5^2 \xRightarrow{r_7} s_1 s_5 \xRightarrow{r_7} s_1^2$$



Extensions: Intermediate initiations

- Initially b^- messages
- then routing ...
- ... but extra z_s^- messages can be generated by s and z_s^+ messages can complete paths at s ...
- b^+ messages reach destination nodes

$$\sum_{r \in R_s} x_r + z_s^+ + b_s^+ = \sum_{r \in R} a_{rs} x_r + z_s^- + b_s^-$$

Extensions: more constraints

Node s initiates b_s^- or more messages and receives b_s^+ or less

$$\sum_{r \in R_s} x_r + z_s^+ + b_s^+ \leq \sum_{r \in R} a_{rs} x_r + z_s^- + b_s^-$$

Node s initiates b_s^- or less messages and receives b_s^+ or more

$$\sum_{r \in R_s} x_r + z_s^+ + b_s^+ \geq \sum_{r \in R} a_{rs} x_r + z_s^- + b_s^-$$

Routes with attributes

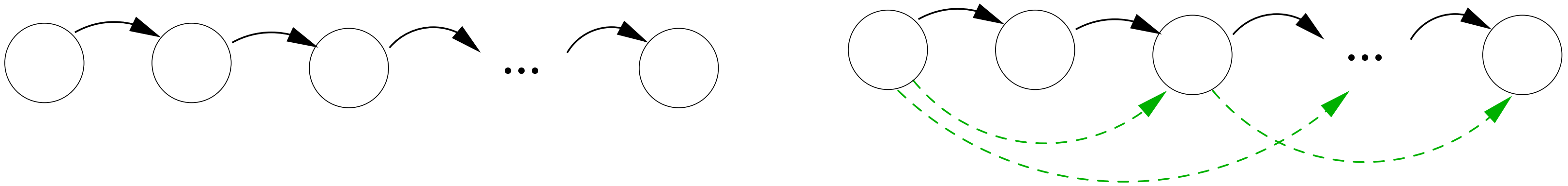
$$\sum_{r \in R} a_{\sigma r} x_r \leq b_\sigma \quad \text{or} \quad \sum_{r \in R} a_{\sigma r} x_r \geq b_\sigma$$

Applications: connectivity

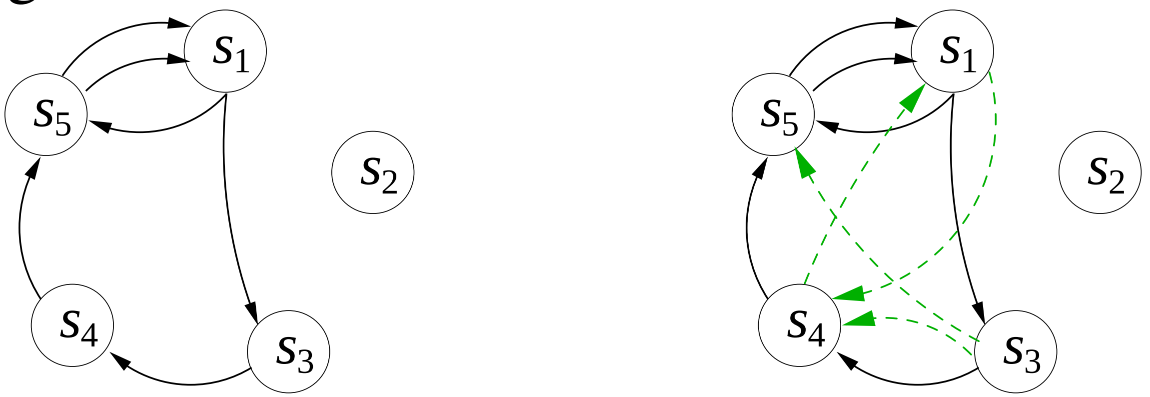
- Basis solutions (Hilbert basis) represent basic routes in a P2P network
- Each basic route $(b^-) \xrightarrow{b} (b^+)$ defines a set of paths for initially b^- messages to travel through the network and finally reach b^+ destinations.
- Any route is a combination of basic ones (non-basic routes have extra circuits)
- Various scenarios by taking b^-, b^+, and b.
  E.g. $b^- = (1, 1, \ldots, 1)$: simultaneous initiation.

Applications: efficiency

- If route h be in active use but long, i.e., $\sum_{r \in R} h_r > L$, then transitive closure (total or partial)



- Clustering



That's the way for s to manage its routing table: $O(1) \leq |T_s| \leq N$

Constant degree $<$ $(\log N)$-degree $<$ $(\sqrt{N})$-degree $<$ full routing state

Applications: reliability

- Sequential and parallel forwarding
- Multiple alternate or independent paths
- Which nodes are most loaded when $M \subset S$ nodes are under attack, i.e. $b_s^- > B \; \forall s \in M$
- What scenarios result in $b_s^+ > B$ for a given node s
- …

Applications: maintenance cost

1. Collection of routing rules
   - available rules
   - essential rules
   - inheritance of data
2. Efficient computation
   - polynomial-time syntactic algorithms
3. Regular nodes and super-nodes
   - regular nodes perform local modeling
   - super-nodes perform global modeling